Discounted Stochastic Games with Voluntary Transfers

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Slides

Discounted Stochastic Games

Natural generalization of infinitely repeated games

- n players
- infinitely many periods, common discount factor $\delta < 1$
- in every period there is a state $x \in X$ (finite)
- Stage game
  - actions $a = (a_1, ..., a_n) \in A(x)$
  - payoffs $\pi(a, x)$
- State can change after every period
  - $\tau(x'|x, a)$: probability that new state is x'
- This talk: Players publicly observe a and x (perfect monitoring)

Example: Cournot Model with Stochastic Reserves

- Two firms $i = 1, 2$ that operate hydro-electric power plants
- State $x = (x_1, x_2) \in \{0, 1, ..., \bar{x}\}^2$ amount of hydro-energy in each firm's water reservoir
- Firma i can sell in a period $a_i \in \{0, 1, ..., x_i\}$ units of energy.
- Stage game profits:

$$\pi_i(a, x) = P(a_1, a_2) a_i$$

- New state depends on random rainfall:

$$x_i' = x_i - a_i + \varepsilon_i$$

Solving stochastic games... for Markov perfect equilibria?

- Most applied literature: Markov perfect equilibria (MPE)
  - actions depend only on current state x

Problems with MPE

- Multiple MPE can exist (e.g. Besanko et. al., 2010)
- Set of MPE payoffs often unknown
- Set of MPE payoffs can be very sensitive to state space
  - single state (infinitely repeated game):
    MPE = repetition of static Nash equilibrium
  - including (almost) payoff irrelevant states, e.g. output in previous period, may allow quite collusive MPE

Solving stochastic games... for subgame perfect equilibria?

- Set of SPE payoffs hard to characterize
- Large discount factors ($\delta \to 1$) & irreducible stochastic game
  - Dutta (1995), Hörner et. al. (2011)
- Fixed δ: Extending algorithms for repeated games?
  - Abreu, Pearce and Stachetti (1990), Judd, Yeltekin, Conklin (2003), Abreu & Sannikov (2011)
- Pareto-optimal equilibria don't have in general a simple structure

This paper

- Considers an economic relevant subset of stochastic games
- Main results:
  - every SPE payoff can be implemented with a simple class of equilibria
  - methods to analytically find or to compute equilibrium payoff sets

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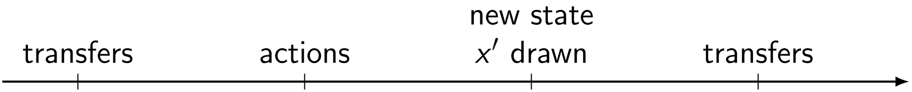
- Considers an economic relevant subset of stochastic games
- Main results:
  - every SPE payoff can be implemented with a simple class of equilibria
  - methods to analytically find or to compute equilibrium payoff sets
- Stochastic games with voluntary monetary transfers and risk-neutral players
  - repeated games: Levin (2003), Goldluecke und Kranz (2010), Malcomson & McLeod (1989), Doornik (2006), Rayo (2007), Klimenko, Ramey and Watson (2008), Harrington and Skrzypacz (2007),...
  - transfers implemented in several cartels via sales between firms

Structure of a period in game with transfers



- Transfers: players chooses simultaneously amount of money they want to transfer to other players
  - no binding liquidity constraints
  - money burning possible
  - received net amount of money will be added to payoffs $\pi_i(a, x)$

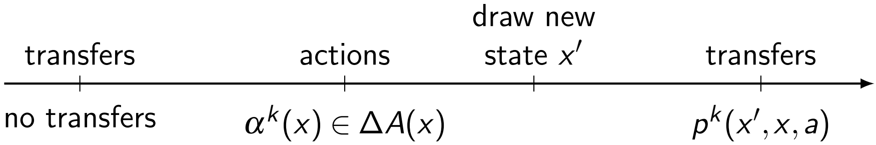
Simple strategy profiles

Basic structure

- $n+1$ phases $k \in \{e, 1, ..., n\}$
  - equilibrium phase $k = e$
  - a punishment phase $k = i$

Simple strategy profiles

Play in phase k state x



exception:
upfront transfers
in first period

Simple strategy profiles

Transition between phases

- phase only changes after upfront transfer or after transfer at end of period
  - player i unilaterally deviates from his transfer $\Rightarrow$punishment phase $k = i$
  - no player unilaterally deviates from transfer $\Rightarrow$equilibrium phase $k = e$
- punishment have a stick-and-carrot structure (similar to Abreu, 1986)

Main Result

Theorem

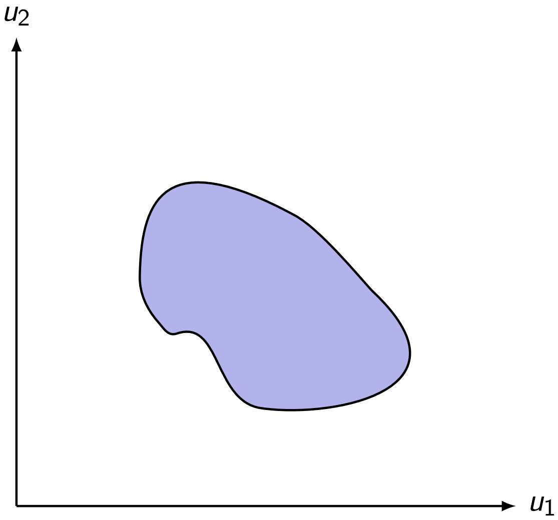
Fix a discount factor δ: Every SPE payoff can be implemented with an equilibrium in simple strategies.

Intuition

Incentive compatible monetary transfers can be used for three important functions

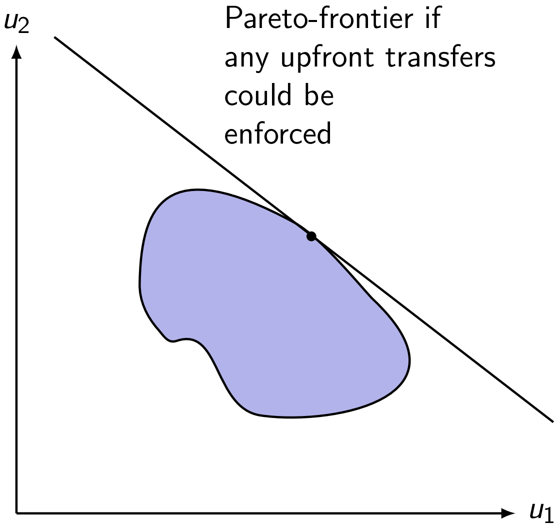
1. Distribute joint payoffs (with upfront payments)
2. Balance incentive constraints between players
3. Fines as punishment

1. Distributing with upfront transfers



blue area =
equilibrium payoffs
without
upfront transfers

1. Distributing with upfront transfers



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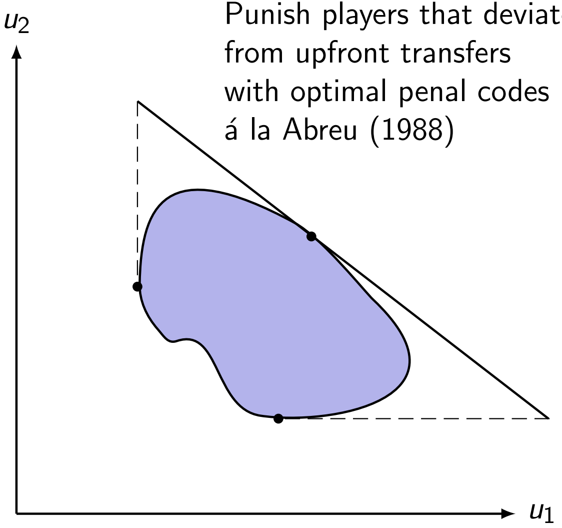

Punish players that deviate
from upfront transfers
with optimal penal codes
á la Abreu (1988)

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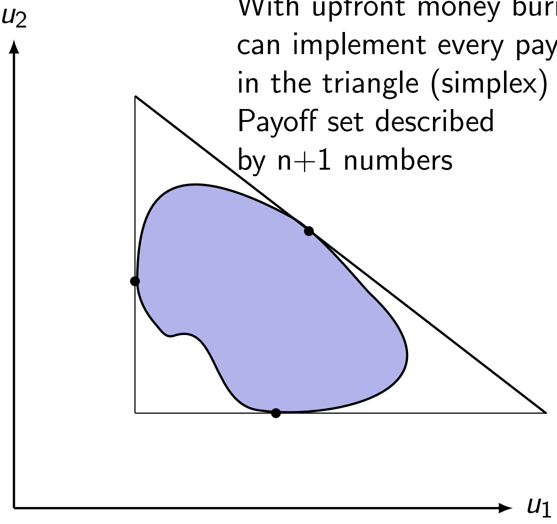
1. Distributing with upfront transfers



With upfront money burning
can implement every payoff
in the triangle (simplex)
Payoff set described
by n+1 numbers

blue area =
equilibrium payoffs
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upfront transfers

2. Balancing incentive constraints

- Repeated asymmetric Prisoners' dilemma
- Aim: (C,C) in every period
- Punishment: (D,D) forever

	C	D
C	3,1	-1,4
D	4,-1	0,0

- Incentive constraints for subgame perfection

$$\text{Player 1: } \sum_{t=0}^{\infty} 3\delta^t = \frac{3}{1-\delta} \geq 4 \Leftrightarrow \delta \geq \frac{1}{4}$$

$$\text{Player 2: } \sum_{t=0}^{\infty} \delta^t = \frac{1}{1-\delta} \geq 4 \Leftrightarrow \delta \geq \frac{3}{4}$$

- Given asymmetries stationary equilibrium play may not be optimal (without transfers)

2. Balancing incentive constraints

- Assume pl. 1 transfers 1 unit of money every period on eq. path to pl. 2. No incentives to deviate from (C, C):

$$\text{Player 1: } \frac{3-1}{1-\delta} \geq 4 \Leftrightarrow \delta \geq \frac{1}{2}$$

$$\text{Player 2: } \frac{1+1}{1-\delta} \geq 4 \Leftrightarrow \delta \geq \frac{1}{2}$$

- Consider summed incentive constraints:

$$\frac{3+1}{1-\delta} \geq 4+4 \Leftrightarrow \delta \geq \frac{1}{2}$$

- General result: if summed incentive constraints hold, one can always find transfers such that no player has incentives to deviate from individual actions or transfers

3. Fines as punishment

- Allow a player who deviates to avoid punishment actions by paying a fine
- Punishment actions only necessary if fines not paid
- After one period of punishment actions, remaining punishment can be settled again with a fine

$\Rightarrow$ Optimal penal codes can be described by one action profile per state (plus transfer / fine scheme)

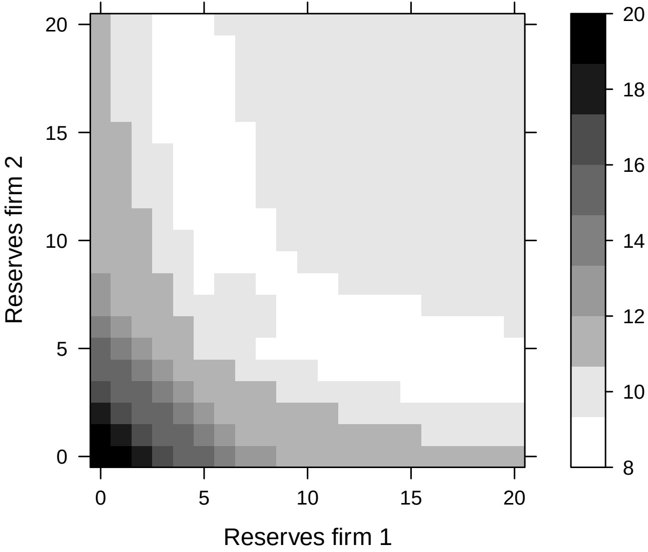
- For mixed action profiles transfer make a player indifferent between all pure actions in the support

Optimal Simple Equilibria & Algorithms

- Paper develops additional results for finding optimal simple equilibria that can implement every SPE payoff by varying upfront transfers.
  - different numerical algorithms
  - guidance to find closed-form solutions

Solving the model of Cournot Competition with Stochastic Reserves...

Prices under Collusion



$\delta = \frac{2}{3}$

$\bar{x} = 20$

$P(a) = 20 - a_1 - a_2$

rain: 3 or 4 units

441 states

$\sim$30 sec. to solve

on my notebook

Principal-Agent Relationship with a Durable Product

- State describes whether principal has a durable product: $x \in \{0,1\}$

$$\pi_P(a,x) = x$$

- Agent can exert costly effort to build or destroy the product $a \in \{-1,0,1\}$

$$\pi_A(a,x) = -c|a|$$

$$x' = \min\{\max\{x+a,0\},1\}$$

- Unique MPE: no transfers, $a(x) = 0$

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'grim-trigger-style" equilibria: after deviation play MPE forever

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'grim-trigger-style" equilibria: after deviation play MPE forever
$\Rightarrow$only zero effort can be implemented

Principal-Agent Relationship with a Durable Product

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$$\pi_P(a,x) = x$$

- Agent can exert costly effort to construction or destroy the product $a \in \{-1,0,1\}$

$$x' = \min\{\max\{x+a,0\},1\}$$

$$\pi_A(a,x) = -c|a|$$

- Unique MPE: $a(x) = 0$, no transfers

Optimal simple equilibria: construction in equilibrium phase and destruction as punishment whenever

$$(1-\delta^2)c \leq \delta^2.$$

Summary

- Allow transfers & assume risk-neutrality in discounted stochastic game
  - Every SPE payoff can be implemented with simple equilibria
  - Algorithms to solve for equilibrium payoff sets
  - Results extend to imperfect monitoring of actions

Useful results to find optimal simple equilibria

- Perfect monitoring, finite action space, set of pure strategy SPE
- There are optimal transfers for given actions $(a^k(x))_{\forall k,x}$

Computing joint equilibrium payoffs and punishment payoffs

$$
\begin{aligned}
U(x|a^e) = &\quad (1-\delta)\Pi(a^e(x),x) &+\delta E[U(x'|a^e)|a^e(x),x] \\
v_i(x|a^i) = &\quad \max_{a_i \in A(x)}\{(1-\delta)\pi_i(a,x) &+\delta E[v_i(x'|a^i)|a_i,a^i_{-i}(x),x]\}
\end{aligned}
$$

$a^k(x)$ can be implemented if and only if

$$
\begin{aligned}
(1-\delta)\Pi(a^k(x),x) \quad &+ \quad \delta E[U(x'|a^e)|a^k(x),x] \geq \\
\sum_{i=1}^{n} \max_{a_i \in A(x)}\{(1-\delta)\pi_i(a_i,a^k_{-i}(x),x) \quad &+ \quad \delta E[v_i(x'|a^i)|a_i,a^k_{-i}(x),x]\}
\end{aligned}
$$

Basic idea of one algorithm

- Assume in round r all action profiles in $A^r(x) \subset A(x)$ can be implemented
- In round $r = 0$ all action profiles can be implemented
- Let

$$U(x|A^r) = \max_{a^e \in A^r} U(x|a^e) \text{ Markov decision process}$$

$$v_i(x|A^r) = \min_{a^i \in A^r} v_i(x|a^i) \text{ Nested Markov decision process}$$

- Let $A^{r+1}(x)$ be all profiles that survive joint incentive constraints given $U(.|A^r)$ and $v_i(x|A^r)$
- Stop once $A^r = A^{r+1}$

Public Correlation and Non-Optimality of Stationary Equilibrium paths

- Stage game:

	A	B
A	0,0	-1,3
B	3,-1	0,0

- Mix between (A, B) and (B, A) $\rightarrow \delta \geq \frac{1}{2}$
- Alternate $\{(A, B), (B, A), (A, B), \ldots\} \rightarrow \delta \geq \frac{1}{3}$